Quiz 5. Multivariable Calculus and Taylor Series

Calculus 3 Section 15. Apr 22nd, 2021 (solution).

1. If $f(x, y, z) = x^2 \tan(3y + 4z)$, then $f_{zyx}(0, -1, 1) = \underline{\ 0\ }$.

2. If $f_x(1, 2) = 3$ and $f_y(1, 2) = -5$, then $f(0.99, 2.02) - f(1, 2)$ is approximately $\underline{\ -0.13\ }$.

3. The graph $z = f(x, y)$ has a tangent plane $z = 6 - 3x - 2y$ at $(1, 1, 1)$. (now we know f_x, f_y)

   Let $g(t) = f(t^3, t^2)$. By Chain Rule, we know $g'(1) = \underline{\ 13\ }$.

4. Let $f(x, y) = (x + y)^2 - 2(x + 1)^2$. There is only one critical point: $\underline{\ (-1, 1)\ }$.

   $f_{xx} = \underline{\ -2\ }$, $f_{yy} = \underline{\ 2\ }$, $f_{xy} = \underline{\ 2\ }$. $D = f_{xx}f_{yy} - (f_{xy})^2$.

   What does the Second Derivative Test say? $\underline{\text{ the critical point is a saddle }}$

5. If $x^2 + y^2 = 4$, what is the extreme values for $f(x, y) = 2x - 3y$?
   Using Lagrange multiplier, we have $2 = \underline{\ 2x\lambda\ }$, $-3 = \underline{\ 2y\lambda\ }$, $x^2 + y^2 = 4$.
   The solution will give us the max and the min.

6. $\displaystyle\int_0^1 \int_1^2 \frac{e^x}{y}\, dy\, dx = \underline{\ (e-1)\ln 2\ }$

7. Change the order of integration.

   $$\int_0^4 \int_{\sqrt{x}}^2 f(x, y)\, dy\, dx = \int_{\underline{0}}^{\underline{2}} \int_{\underline{0}}^{\underline{y^2}} f(x, y)\, dx\, dy$$

8. Change into polar coordinates.

   $$\int_{-2}^2 \int_0^{\sqrt{4-y^2}} \sqrt{x^2 + y^2}\, dx\, dy = \int_{\underline{-\pi/2}}^{\underline{\pi/2}} \int_{\underline{0}}^{\underline{2}} \underline{\ r^2\ }\, dr\, d\theta$$

9. Let $u = 2x + y$ and $v = x + 3y$. The Jacobian of this transformation is $\dfrac{\partial(x, y)}{\partial(u, v)} = \underline{\ 1/5\ }$.

10. $xe^x = x + x^2 + \underline{\dfrac{x^3}{2!}} + \underline{\dfrac{x^4}{3!}} + \dfrac{x^5}{4!} + \cdots$

    $$\int_0^x \tan^{-1} t\, dt = \frac{x^2}{2} - \frac{x^4}{12} + \frac{x^6}{\underline{30}} - \frac{x^8}{\underline{56}} + \cdots$$